

FRACTAL DIMENSION OF VISCOUS FINGERING PATTERN FROM TWO VISCOUS OILS

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ABSTRACT

We study the evolution of fingering pattern with two different high viscosity gear oils in planar Hele Shaw cell. Fingering being instantaneous process, it is recorded using digital camera and movie frames at selected time of one second were extracted. The images so obtained were further processed for extraction of the border to be used for determination of fractal dimension using Richardson's plot technique. For implementation of the box counting technique the complete image of the fingering pattern was used. Richardson's plot technique is effective in finding out the complexity of structure and texture associated with ramified boundaries where as box counting technique provides information on complexity of gross shape and structure. It is interesting to note that the shapes of fingering patterns have different degree of complexity of structure and texture at different length scales. This is clearly revealed by Richardson's plot technique where the fractal dimensions at lower and higher length scale (λ) are appreciably different. Details are presented and results discussed.

KEYWORDS: Fractal, Hele Shaw Cell, Viscous Fingering, Box Counting, Richardson Plot

INTRODUCTION

Fractal patterns¹ characterize the non-equilibrium growth, such as the electro-deposition,² viscous fingering,³ dendritic shape of snowflakes,⁴ the shape of bacteria colonies growing in stressed environments,⁵ and dielectric breakdown⁶ etc. All these growth processes lead to structures that are complex in shape and the parameters governing shape are large in number.

When a less viscous fluid is forced into a more viscous fluid under pressure, the interface between the two becomes unstable and the less viscous fluid enters by force into the more viscous "defending" fluid which leads to the formation of a finger-like pattern known as "viscous fingers"⁷ which may repeatedly branch and sometimes form a fractal pattern. Under appropriate approximations the Laplace equation can express the interface, as in diffusion-limited processes with suitable boundary conditions

$$\nabla^2 u(x, t) = 0 \quad (1)$$

Viscous fingering is first observed by petroleum engineers when an aqueous solution displaces more viscous oil in underground reservoirs. During secondary oil recovery,⁸ the viscous fingering phenomenon is important in problems such as fluid flow in porous media,^{9,10} dendritic solidification, combustion in two dimensions¹¹ and electrochemical deposition.¹² Extensive research efforts have been undertaken by workers in the field to understand the various physical factors governing viscous fingering.

VISCOUS FINGERING IN HELE SHAW CELL

We used Hele Shaw Cell whose construction discuss¹³. In this construction we used two different viscous thick oil as high viscosity fluid and air as low viscosity fluid.

More viscous oil or fluid is displaced by the less viscous fluid at different pressure. The gap between the plates filled with high viscous fluid and upper plate clamp for any displacement due to injection of low viscous fluid under different pressure.

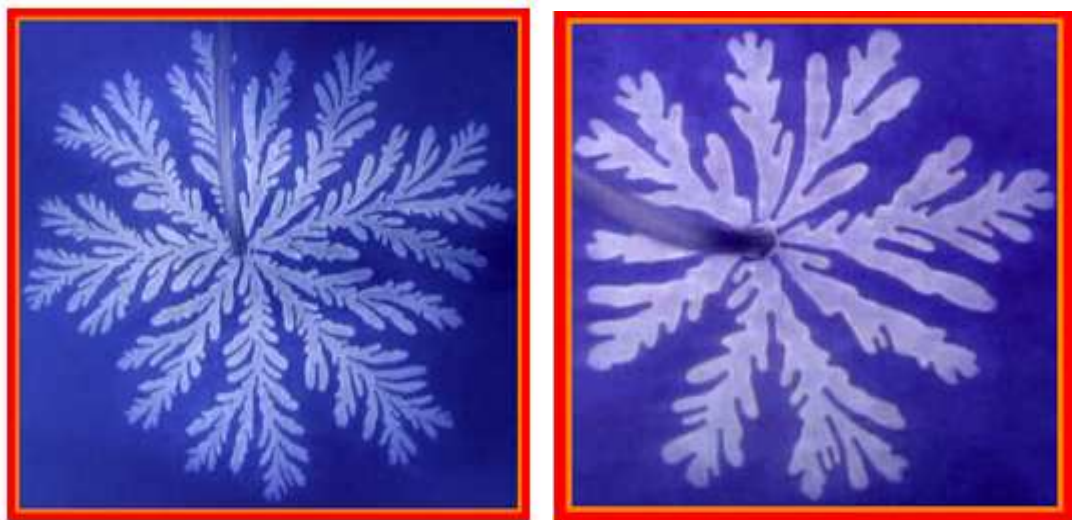
As the fingering over within fraction of seconds, we have used video recording instead of “still” photography with help of illuminated diffuse light digital camera. Then separated the frame for the finer details of the evolution and selected the frame at the instant of 1 sec for different pressure.

The fingering patterns obtained using two different viscous thick oil as a more viscous fluid and air as a less viscous fluid under different pressure at time $t=1$ second as shown in figure 1a-1f.

To study the characterization of the patterns in terms of fractal dimension of different viscous oil at different pressure, we analyzed the fingering patterns using the box counting techniques. The box counting technique was implemented using the images with computer program.

The box counting dimensions of figure 2a-2 b are 1.703 and 1.685 for two different thick oil at high pressure for solid pattern. The dimensions are also given for the moderate pressure and low pressure for the different oils for solid as well as contour pattern in the Table 1. The figure 2a-2e and figure 3a-3e shows the plot of $\log(N)$ against the $\log(r)$ for the two cases. The data points lie along a straight line; least square fit is also shown.

Figure 1a-1e also analyzed by using Richardson's plot technique for fractal dimension. From comparison of the fractal dimensions on the two length scale of longer and shorter length, the six patterns discussed in using plot of $\log(N)$ against $\log(\lambda)$ from figure 3a-3e and summary of them is given in Table 2.



a) High Pressure –Low Viscous

b) High Pressure –High Viscous



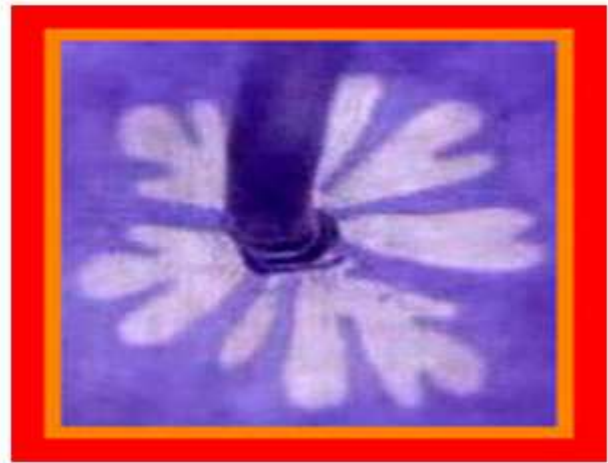
c) Moderate Pressure –Low Viscous



d) Moderate Pressure-High Viscous

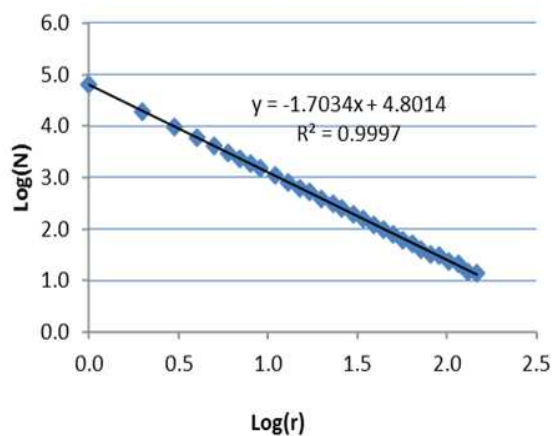


f) Low Pressure-Low Viscous

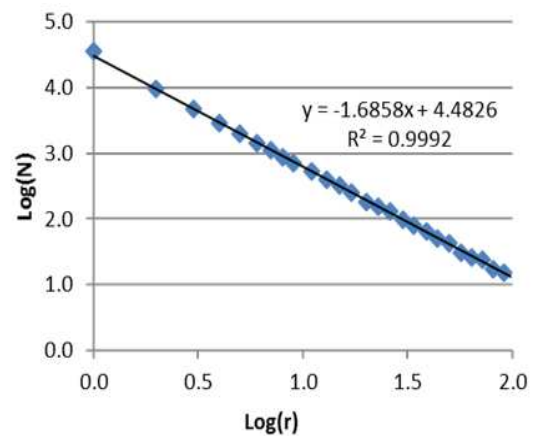


e) Low Pressure-High Viscous

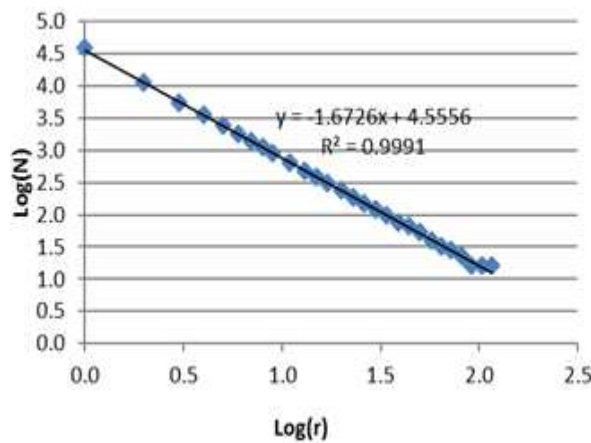
Figure 1: The Fingering Patterns Obtained Using Two Different Viscosity Thick Oil and Air as Less Viscous Fluid Under Different Pressure Conditions at Time t=1sec



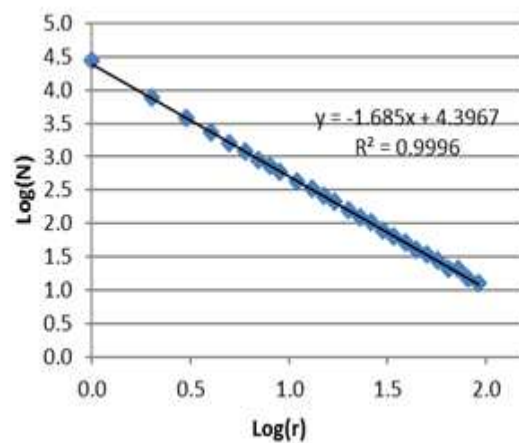
a) High Pressure –Low Viscous



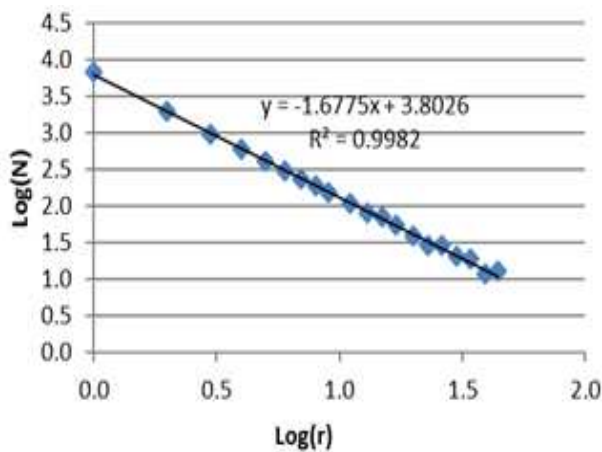
b) High Pressure –High Viscous



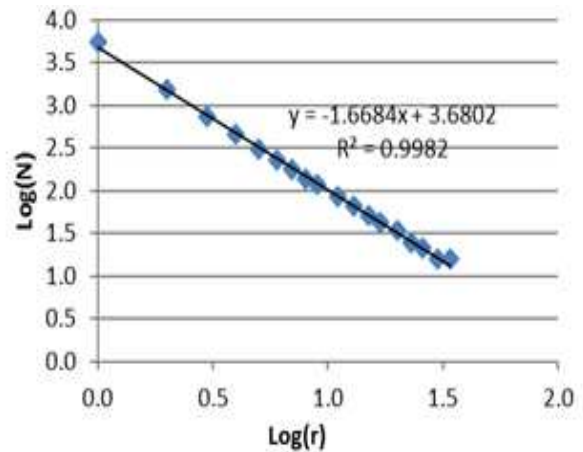
c) Moderate Pressure-Low Viscous



d) Moderate Pressure-High Viscous

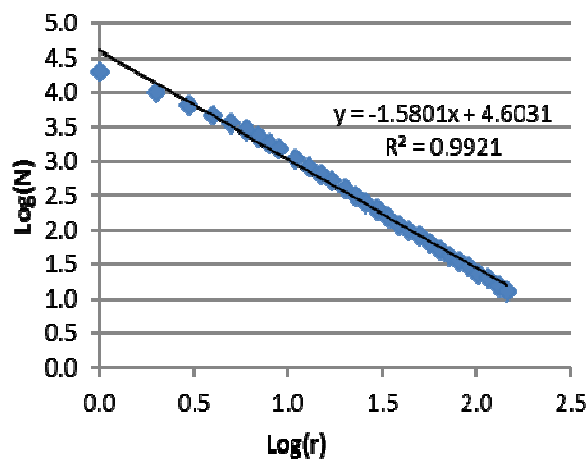


e) Low Pressure-Low Viscous

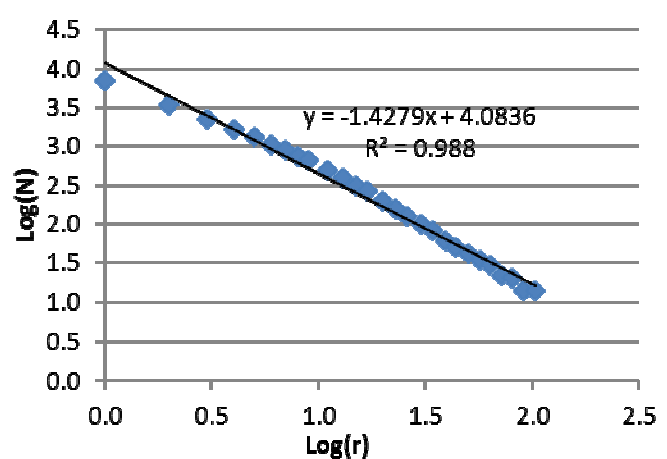


f) Low Pressure-High Viscous

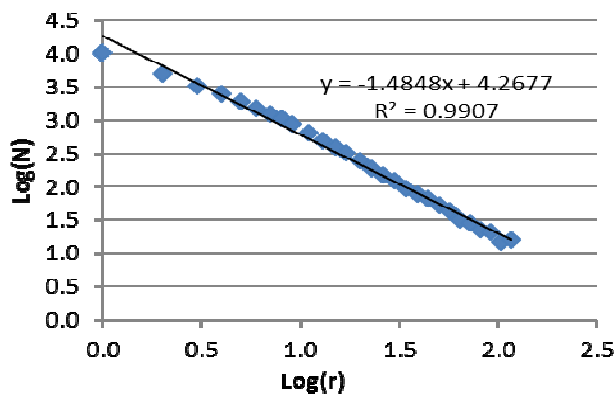
Figure 2: The Graph of Box Counting of Different Pressure for Two Different High Viscous Fluid for Solid Patter at Time $t=1\text{sec}$



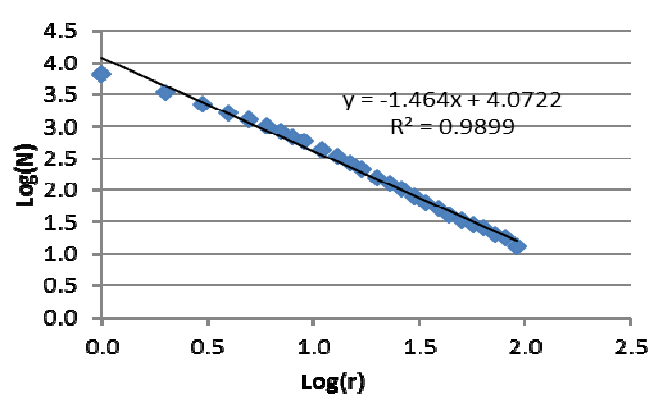
a) High Pressure -Low Viscous



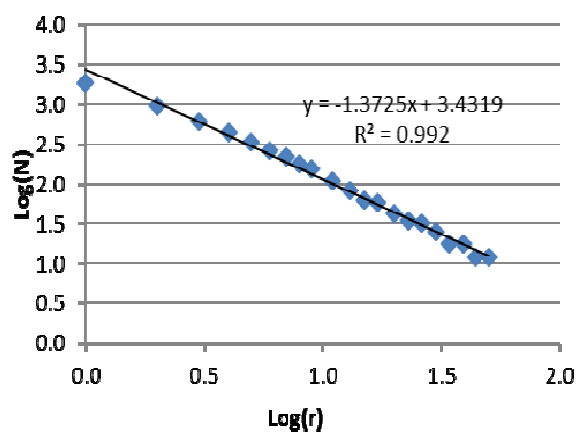
b) High Pressure -High Viscous



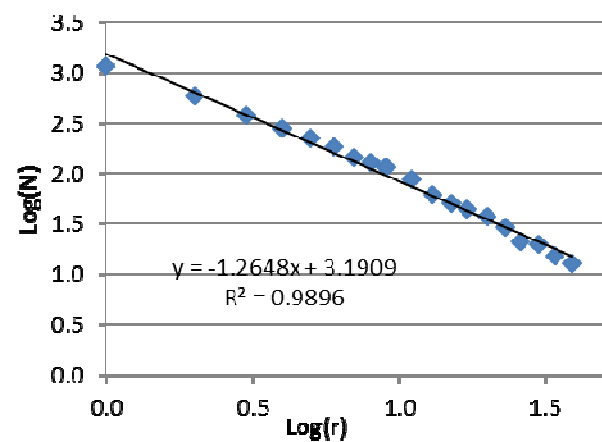
c) Moderate Pressure-Low Viscous



d) Moderate Pressure-High Viscous

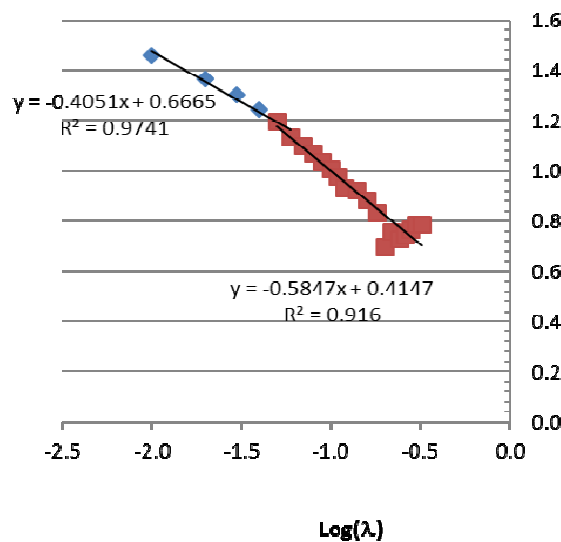


e) Low Pressure-Low Viscous

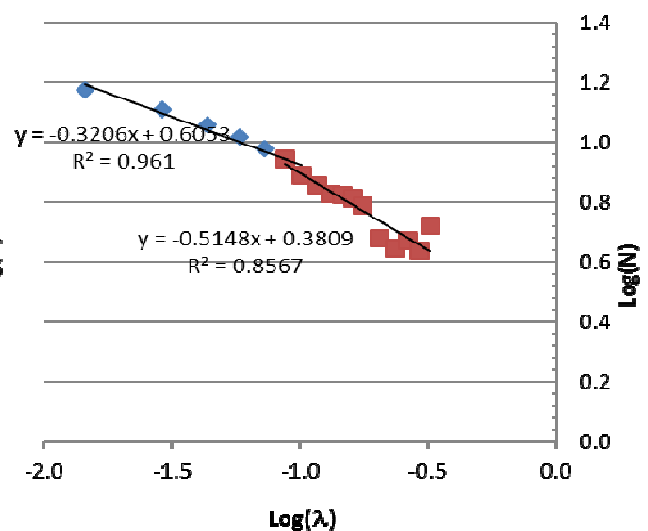


f) Low Pressure-High Viscous

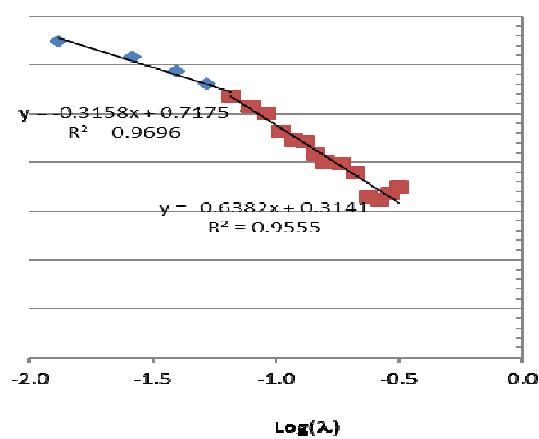
Figure 3: The Graph of Box Counting of Different Pressure for Two Different High Viscous Fluid for Border Pattern at Time $t=1\text{sec}$



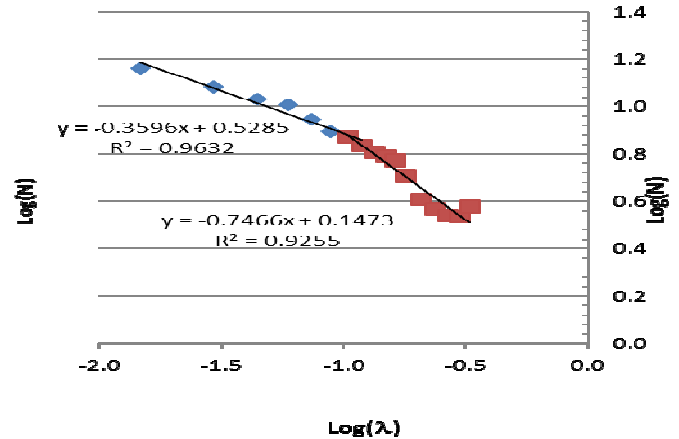
a) High Pressure -Low Viscous



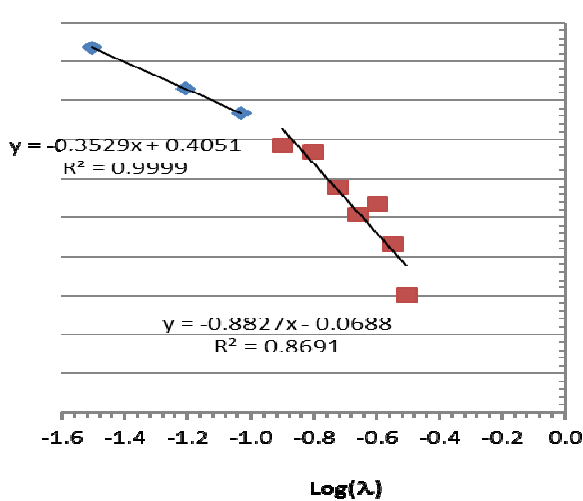
b) High Pressure -High Viscous



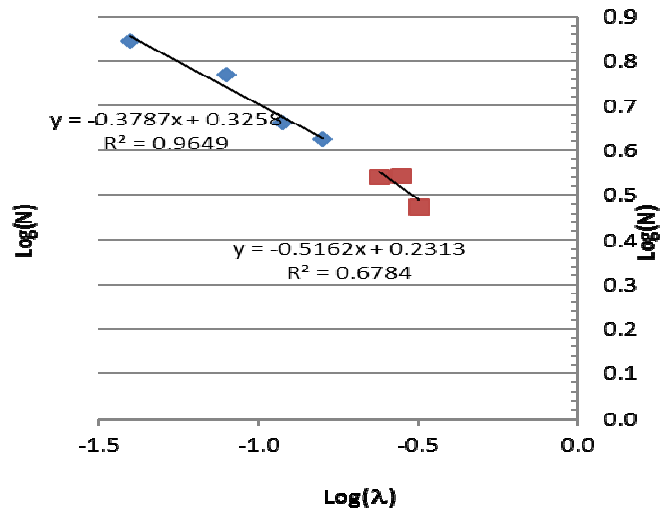
c) Moderate Pressure-Low Viscous



d) Moderate Pressure-High Viscous



e) Low Pressure-Low Viscous



f) Low Pressure-High Viscous

Figure 4: The Graph of Richardson's Plot of Different Pressure for two different High Viscous Fluids for Contour Pattern at Time t=1sec

Table 1: The Summary of the Fractal Dimensions Obtained Using the Box Counting Technique For Different Pressures for Two Different Oils at t=1sec

Fractal Dimension						
Pressure	Solid			Contour		
	Figure. No	Less Viscous Oil	More Viscous Oil	Figure. No	Less Viscous Oil	More Viscous Oil
High Pressure	1a-1b & 2a -2b	1.703	1.685	1a-1b & 3a- 3b	1.580	1.427
Moderate pressure	1c-1d & 2c-2d	1.672	1.685	1c-1d & 3c-3d	1.484	1.464
Low Pressure	1e-1f & 2e -2f	1.677	1.668	1e-1f & 3e -3f	1.264	1.264

Table 2: The Summary of the Fractal Dimensions Obtained Using the Richardson's Plot Technique Different Pressures for Two Different Oils at $t=1\text{sec}$ Where λ is Feret's Diameter

Pressure	Fractal Dimension					
	Less Viscous Oil			More Viscous Oil		
	Figure. No	Low λ	High λ	Figure. No	Low λ	High λ
High Pressure	1a & 4a	1.352	1.882	1b & 4b	1.378	1.516
Moderate pressure	1c & 4c	1.315	1.638	1d & 4d	1.359	1.746
Low Pressure	1e & 4e	1.405	1.584	1f & 4f	1.320	1.514

CONCLUSIONS

Viscous fingering studied using two different viscous oil as more viscous fluid and air as less viscous fluid under different pressure at time $t=1\text{sec}$ in Hele Shaw cell. In box counting technique it is observed that as the pressure increases fractal dimension also increases for both the oils in solid as well as contour pattern but for the comparison of two viscous oil, it is observed that more viscous oil has less fractal dimension than less viscous oil. It is also observed that the less viscous oil has finer fingering pattern at all pressure than more viscous oil.

In Richardson's plot technique it is observed that the fractal dimension at shorter length scale is less than that at larger length scale. This shows that complexity of structure is greater than complexity of texture. This is an agreement with the expected from a pattern with wider branches and very limited finer branching. It is also observed that fractal dimension at moderate pressure for lower and larger scale is more for more viscous oil than less viscous oil while for other pressure there is some uncertainty in the result of fractal dimension.

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